1. If a snowball melts in such a way that the change in its radius is proportional to its surface area ($4\pi r^2$) and its initial radius is 3" and its radius after an hour is 2" when will its radius be 1"? 0"?

$dr/dt = -4kr^2 \Rightarrow (1/r^2)dr/dt = -4k \Rightarrow \int (1/r^2)dr/dt\, dt = \int -4k\, dt \Rightarrow -1/r = -4kt + C \Rightarrow r = 1/(C+4kt)$
$r(0) = 3$ so $C = 1/3$. $r=3/(1+12kt)$ $r(1) = 2 = 3/(1+12k) \Rightarrow 2+24k = 3$; $24k = 1$; $k = 1/24$. Thus $r=6/(2+t)$.
Then $r=1=6/(2+4)$ after 4 hours. The radius will technically never be 0 since $6/(2+t) > 0$ for all $t \geq 0$.

2. Sammy is a member of a rare, exotic endangered aquatic species who resides at Sea World in Florida. Unfortunately, until recently Sammy was not adequately cared for. Someone allowed the saline concentration in his 50 gallon tank to rise to a dangerous 5%. The new manager (remember her) doesn't want to shock Sammy's system by correcting the problem all at once (exotic aquatic species have delicate constitutions) so she plans to continuously add 4% saline solution at a rate of 10 gallons per day while bleeding off tank water at the same rate. Assuming Sammy keeps his tank well stirred, how long will it take to bring the saline level to a more reasonable 4.5%? Mathematically speaking, how long would it take until the level was 4%?
$dA/dt = (.04)(10) - (A/50)(10) = .4-A/5 = (2-A)/5$ so $dA/(A-2) = -dt/5$ and $\ln(A-2) = -t/5 + C$ so $A-2= Ce^{-t/5}$
$A = Ce^{-t/5} +2$. $A(0) = (.05)(50)=2.5$ so $C = 0.5$. If the concentration is 4.5 then $A = 4.5(50) = 2.25$
so $.5 e^{-t/5} = .25 \Rightarrow e^{-t/5} = .25/.5 = 1/2$ $t = 5\ln(2) \approx$ or approximately 3 days 11 hours and 10 minutes.
Mathematically speaking it would never quite reach 4%.

3. It's March of 2020 and the President has just received a communication from the crew of the *Hippocrates*, on the first live expedition to Mars. The purpose of this emergency mission is to locate and bring back to earth a supply of a new radioactive isotope, Goldinite discovered by a previous, unpeopled expedition. Scientists believe that a sufficient quantity of Goldinite in a reaction with Earth-native carbon will produce enough of a new drug to eradicate a newly discovered deadly communicable disease called R.T.W.D. Unfortunately, the sample of Goldinite retrieved by the previous mission wasn't large enough for the scientists to properly study. Although they know it is radioactive they were unable to determine it's half-life. The astronauts have located a source of Goldinite. They brought 20 lbs to the ship's laboratory and after one day 12 lbs of goldinite remained, the other 8 lbs had decayed into the inert new element Drossite. The scientists back on earth need 40 lbs to eradicate the disease. It takes time to mine the Goldinite and in this case time isn't money; it's human lives back on earth being lost to RTWD. If the return trip to Earth will take 20 days, how much Goldinite must they have on board at lift off in order to have the necessary 40 lbs left when they touch down on Earth? The payload limit on the Hippocrates is 1000 tons (1 ton is 2000 lbs); what did the message say?
$dA/dt = -kA$ so $A(t) = A_0e^{-kt}$ so $20e^{-k} = 12$. So $k = \ln(20/12) \approx .51$. $A_0e^{-.51(20)} = 40$ so $A_0 = 40\, e^{.51(20)} = 1{,}076{,}130$ lbs. The message would have said we can bring enough home with us so, we're mining and then will be on our way.

4. Michael, a stressed out Differential Equations student is having a nightmare in which he is trapped in a cubical room which, although it remains perfectly cubical is shrinking in such a way that the rate of change of the volume of the room is always proportional to its height. The room was originally 10'x10'x10'. After one hour it was 8'x8'x8'. If Michael doesn't wake up, when will his room disappear completely?

$V = h^3$ so $dV/dt = 3h^2dh/dt = -kh$. So $3h\, dh/dt = -k$ and $(3/2)h^2 = C - kt$ or $h^2 = C - 2kt/3$. $10^2 = C$.
$8^2 = 100-2k/3$. So $2k/3 = 36$; $k=54$. So $h=0$ when $36t=100$ or $t = 100/36 \approx 2.78$ hours or just under 2 hours and 47 minutes.

5. Batman and Robin have just discovered that the water supply of Gotham city has been sabotaged by a cast of circus characters (the Joker, the Riddler, Cat Woman, etc.). The water supply consists of a 30 ton primary reservoir from which the good citizens of Gotham drain water at a pretty constant rate of 1 ton a day. During the dry season (which is right now) there is a gigantic pump which feeds water into the primary reservoir from a secondary reservoir at a constant rate of 1 ton a day, to offset the drain. The saboteurs damaged the pump so that the reservoir dwindled to a dangerously low 20 tons. In addition, the saboteurs doctored the water so that the 20 tons now remaining is 5% sidite. Sidite is a mind controlling drug the

saboteurs have been using to influence the citizens of Gotham; so long as the water the citizens are drinking contains 4% or more sidite, the saboteurs will retain control of the city. Batman and Robin have managed to restore pump activity and are adding clean water from the secondary reservoir at a rate of 2 tons a day. What will the concentration of sidite be when the reservoir is full again? How long until the city will be free of the saboteur's control?

$dS/dt = -S/(20 + t)$. So $(20 + t)dS/dt - S = 0$ or $(20+t) S = C$. $S(0) = (20)(.05) = 1 = C/20$ so $c=20$. $S = 20/(20+t)$. The reservoir will be full when $t = 10$ and $S = 20/(30) = 2/3$. The concentration at that time will be 2/90 or 2.22...%. Concentration at time t is $20/(20+t)^2 = .04$ when $(20+t)^2 = 500$ or $t = 2.3607$. That is 2 days 8 hours 39 minute and 25 seconds until the city is free.

6. When adding water to fill a fishtank whose water level is low, you are supposed to remove an amount equal to that by which it is low then add twice that amount of fresh water. Suppose the new manager of Sea World notices that Shamu's 400,000 ton tank is 20,000 tons low. (ie. there's only 380,000 tons of water in the tank) Also suppose that she decides to fill the tank by adding 50 tons per hour of a 3% salt water solution and bleeding off 25 tons per hour of the water in the tank. She will stop the process when the tank is filled. If the concentration of salt in the tank is initially 5% model the salt in the tank. Assume that Shamu keeps the tank well stirred. What is the concentration of salt at time t? when the tank is filled? What concentration of saline should she add if she desires a final concentration of 4.6%?

$dA/dt = 50(.03)-25A/(380,000+25t)$; $A(0) = 380000(.05) = 19000$;

$dA/dt =1.5-A/(15,200+t) \Rightarrow dA/dt + A/(15,200+t) = 1.5 \Rightarrow (15,200+t)dA/dt + A = (15,200+t)(1.5) =1.5t+22800$

So $(15,200+t)A = .75t^2 +22800t + C$ where $(15,200)(19000) = C = 288,800,000$

$A(t) = (.75t^2 +22800t +288,800,000)/(15,200+t)$. Concentration is $A(t)/(380,000+25t)$

$=(.75t^2 +22800t +288,800,000)/[(15,200+t)(380,000+25t)]=(.3t^2 + 912 t + 11,552,000)/[152,00+t)^2]$ The tank is filled when $t = 20,000/25 = 800$. So the concentration is .048725.

If she used 100k% solution instead of 3% then:

$dA/dt = 50(k)-25A/(380,000+25t)$;

$dA/dt =50k-A/(15,200+t) \Rightarrow dA/dt + A/(15,200+t) = 50k \Rightarrow (15,200+t)dA/dt + A=(15,200+t)(50k)$
$=50kt+760000k$

So $(15,200+t)A = 25kt^2 +760,000kt + C$ where $(15,200)(19000) = C = 288,800,000$

$A(t) = (25kt^2 +760,000kt +288,800,000)/(15,200+t)$. Concentration is $A(t)/(380,000+25t)$

$=(25kt^2 +760,000kt +288,800,000)[(15,200+t)(380,000+25t)]=(kt^2 + 30400k t + 11,552,000)/[152,00+t)^2]$ And the concetration when the tank is filled is $(1805+138k)/40,000 = .046$ if $k = 0.253623$. So she would want to use 2.5% OR 2.6 % solution depending on whether she wanted it a little less or a little more salty than desired.